A Detailed List and a Periodic Table of Set Classes

Luis Nuño

Communications Department, Polytechnic University of Valencia, Valencia, Spain

Dpto. Comunicaciones, UPV, Camino de Vera S/N, 46022 – Valencia, Spain

lnuno@dcom.upv.es, harmonicwheel@gmail.com

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Abstract

In this paper, pitch-class sets are analyzed in terms of their intervallic structures and those related by transposition are called a set type. Then, non-inversionally-symmetrical set classes are split into two set types related by inversion. As a higher version of the interval-class vector, I introduce the trichord-type vector, whose elements are the number of times each trichord type is contained in a set type, as well as a trichord-class vector for set classes. By using the interval-class, trichord-class, and trichord-type vectors, a list of set classes and types is developed, including, apart from the usual information, the intervallic structures and the trichord-type vectors. The inclusion of this last characteristic is the most significant difference with respect to previously published lists of set classes. Finally, a compact periodic table containing all set classes is given, showing their main characteristics and relationships at a glance.

Keywords: list of set classes, interval-class vector, trichord-class vector, trichord-type vector, periodic table
1. Introduction

Pitch-class set theory or, in a broad sense, post-tonal theory, has been consolidated during the second half of the twentieth century and has proved to be a powerful tool for composing and analyzing atonal music, and can be applied to tonal music as well. Babbitt (1955, 1960, 1961), Lewin (1959, 1960), Martino (1961) and Perle (1991 [1962]) developed the main concepts and applied them to twelve-tone composition. Forte (1973) introduced his set-class names and considered the pitch-class sets in a general context, not necessarily in twelve-tone music. Rahn (1980) rewrote much of this theory in a more formal and mathematical style and provided additional results. Further mathematical developments were given by Lewin (2011 [1987]) and Morris (1987, 2001). Additionally, an overview of mathematics involved in the development of twelve-tone system is given by Morris (2007).

As a brief summary, pitch-class sets related by transposition or inversion are grouped into a set class. We represent every set class by a simple form, give it a name, and determine its symmetries and complement. Additionally, the interval-class vector lists the interval classes contained in each set class, which characterizes, to a great extent, its sonority (but not completely!). Lists of set classes including all that information are an essential part of this theory and are available in many texts.

In this paper, the basic concepts are introduced through group theory and, particularly, group actions. This way, pitch classes are considered as elements of the set of integers modulo 12, and all pitch-class sets related by transposition, here called a set type, are defined as the orbit of a pitch-class set under the action of the group of transpositions. Then, I represent a set type by the so-called intervallic form or IF (the intervals between every two adjacent pitch classes, including the interval between the last and the first ones), so that we can easily derive its inversion and its complement. Furthermore, each set class not being inversionally symmetrical is split into two set types related by inversion, which allows distinguishing, for example, between major and minor triads, or between dominant and half-diminished seventh chords.

As a higher version of the interval-class vector (ICV), I introduce the so-called trichord-type vector (TTV), whose elements are the number of times each trichord type is
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contained in a set type. I find that the TTV fully characterizes the sonority of a set type, except in only one case. As well, when dealing with set classes instead of set types, I obtain a reduced version of the TTV, which is here called the trichord-class vector (TCV). It is easily obtained from the TTV and, in contrast to the ICV, fully characterizes a set class.

Then, I provide a detailed list of set classes and types, which adds the IF and the TTV to similar lists published in Forte (1973), Rahn (1980) or Straus (2016). Finally, I develop a compact periodic table including all set classes, which shows their main characteristics and relationships at a glance.

There are many texts and articles that provide full explanation of the basic concepts and terminology used in this paper, Straus (1991) being a primer and Straus (2016) an undergraduate textbook. However, there are sometimes slight differences among the terms and notations used by different authors. For this reason, sections 2 and 3 give a summary of those concepts and terms, along with the notations and acronyms here used. Section 2 also gives the definition of the IF, together with its relevant properties.

Section 4 deals with the TTV, including the formulas relating the TTVs of a set type and its complement, although they are derived in Appendix 1. The TCV is also introduced here. Section 5 describes the detailed list of set classes and types, which is given in Appendix 2. And section 6 includes the above-mentioned periodic table. The ICV, TCV and TTV played a crucial role in arranging the set classes and types, both in the detailed list and the periodic table.

2. Intervallic Form

The 12 pitch classes are represented as elements of the set of integers modulo 12, \( \mathbb{Z}_{12} \), where 0 corresponds to note C and 11 to note B. In this study, integers 10, 11 and 12 are represented by letters A, B and C, respectively (C is only used in one line in Appendix 2). A pitch-class set is then a subset of \( \mathbb{Z}_{12} \) and is written in brackets without commas; for example, \([95A24]\).
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Following Crans, Fiore, & Satyendra (2009), I consider the cyclic group of transpositions $T$ of order 12 and the dihedral group of transpositions and inversions $T/I$ of order 24. Then, a *set type* is here defined as the orbit of a pitch-class set under the action of the cyclic group $T$, whereas a *set class* is the orbit of a pitch-class set under the action of the dihedral group $T/I$. Since the inversion of a pitch-class set also gives rise to a set type, a set class is formed by a set type and its inversion.

For a pitch-class set, its *degree of transpositional symmetry* $s_T$ is the order of its stabilizer in the cyclic group $T$; and, similarly, we can define its *degree of dihedral symmetry* $s_{T/I}$ as the order of its stabilizer in the dihedral group $T/I$. Therefore, from the orbit-stabilizer theorem, the number of different pitch-class sets in a set type is $12/s_T$ and the number of those in a set class, $24/s_{T/I}$. In some references (as Straus 2016), the *degree of inversional symmetry* $s_I$ is also defined, which is related to the previous ones by $s_I = s_{T/I} - s_T$ (or $s_{T/I} = s_T + s_I$). The value of $s_T$ must be a divisor of 12 and $s_{T/I}$ is equal either to $s_T$ ($s_I = 0$) or $2s_T$ ($s_I = s_T$), in which case the pitch-class set is said to be *inversionally symmetrical*. All pitch-class sets in a set class, as well as their complements, have the same values of $s_T$ and $s_{T/I}$ (and, therefore, $s_I$).

For simplicity, it is common to write pitch-class sets and set types in their *normal forms*. There are, however, two widely used normal forms, one given by Forte (1973) and the other by Rahn (1980), and they are not always equal. In each case, the lesser of the normal forms of a set type and its inversion, with respect to the lexicographic order, is the corresponding *prime form*. Unless otherwise indicated, the lexicographic order will be assumed when comparing normal forms or vectors (see below).

In this study, another normal form, together with its corresponding prime form, are considered. Given a pitch-class set, such as $[95A24]$, the pitch classes are first written in ascending order within an octave, for example $[59A24]$, and then the differences from each integer to the next one are obtained, which gives $\{41421\}$ (the last integer being the difference from the last to the first integer in the pitch-class set). This result, or any of its circular shifts, is here called the *intervallic form* (IF) and will be written in braces. It represents the set type of the given pitch-class set. Since an IF contains the sequence of ordered pitch-class intervals in a pitch-class set, the sum of all integers in any IF is
always 12. The least of the circular shifts of an IF (with respect to the lexicographic order) will be the *normal intervallic form*. In this example, it is \{14142\}. The IF is the same as the “interval notation” given by Regener (1974), except that he chooses as normal form the greatest in lexicographic order and does not consider a prime form, which may cause difficulties for relating the two set types of a set class when arranging them. The IF has several important properties:

1) Given a set type, the IF easily allows obtaining both the Forte and Rahn normal forms. The latter corresponds to the circular shift of the IF which is the greatest in colexicographic order. In this example, it is \{21414\}, thus giving the pitch-class set starting from 0: [02378], which is actually the Rahn normal form. With respect to the Forte normal form, it corresponds to the circular shift of the IF which has the maximum integer on the right and, if there are several options (as in this example, where there are two 4’s), the least in lexicographic order. In this example, it is \{14214\}, thus giving the pitch-class set starting from 0: [01578], which is actually the Forte normal form. Although in this example the Forte and Rahn normal forms are different, in most cases they are the same.

2) Contrary to Forte and Rahn normal forms, the IF of a pitch-class set allows obtaining the IF of its inversion directly: it is just the same IF but in reverse order. So, in this example, it is \{12414\}, which is also the normal IF. Therefore, its corresponding Rahn normal form is obtained from the circular shift \{14124\} (the greatest in colexicographic order), which gives the pitch-class set starting from 0: [01568]. And its Forte normal form is obtained directly from \{12414\} (because it already has the maximum integer on the right and is the least circular shift in lexicographic order), which gives the pitch-class set starting from 0: [01378]. Given the normal IFs of a set type and its inversion, the lesser will be the *prime intervallic form* of the corresponding set class. In this example, it is \{12414\}. Normally, as in this example, it also corresponds to the Forte and Rahn prime forms, but this is not always true.

3) Contrary to Forte and Rahn normal forms, the IF of a pitch-class set easily allows obtaining the IF of its complement, too. For example, the IF of [59A24] is \{41421\}, which can be mentally represented as in Figure 1-a). Its complement is shown in Figure 1-b), whose IF will be \{1131123\} or any of its circular shifts.
To sum up, for a set type, it is simple to obtain the IF, as well as the IF of its inversion, its complement and, consequently, the inversion of its complement (which is equal to the complement of its inversion). And these IFs easily allow obtaining the corresponding Forte and Rahn normal forms. On the contrary, obtaining all these normal forms from one of them, either Forte or Rahn, without using the IF, is laborious. Additionally, the IF easily allows determining whether or not a set class is inversionally symmetrical (by comparing the IF in both directions), as well as obtaining its degree of transpositional symmetry (which is equal to the number of periods in the IF, analyzed as a periodic structure). In short, the IF is a simple and versatile representation of set types.

3. Interval-Class Vector

The number of pitch classes in a pitch-class set or a set class is its cardinality and will be represented by $c$. For $c = 2$, there are 6 different set classes: the dyads, also called the unordered pitch-class intervals or interval classes, which will be arranged by increasing prime IF. All of them are inversionally symmetrical and have $s_I = s_T = 1$, except \{66\}, the tritone, which has $s_I = s_T = 2$.

For $c > 2$, the interval-class vector (ICV) lists the number of times each of the 6 dyads is contained in a given set class. This characterizes, to a great extent, the sonority of the set class, but not completely. For a set class with cardinality $c$, the sum of the elements of the ICV is $\binom{c}{2} = c(c - 1)/2$. The ICVs of set classes are given in many sources, as well as in Appendix 2.
The ICV is defined for the action of the dihedral group T/I, so all pitch-class sets forming a set class have the same ICV. However, there are some different set classes with the same ICV. They are said to be Z-related and no more than 2 different set classes have the same ICV (this is true in \( \mathbb{Z}_{12} \), but for a study of the Z-relation in \( \mathbb{Z}_n \) see Mandereau et al. 2011). On the other hand, there is a simple and well-known relationship between the ICVs of a set class and its complement, derived in Appendix 1. Given a set class with cardinality \( \mathcal{C} \) and ICV \([d_1 \cdots d_6]\), its complement will have the cardinality \( \mathcal{C}' = 12 - \mathcal{C} \) and ICV \([d'_1 \cdots d'_6]\), which can be obtained by

\[
\begin{bmatrix}
  d'_1 \\
  d'_2 \\
  d'_3 \\
  d'_4 \\
  d'_5 \\
  d'_6 \\
\end{bmatrix} = \begin{bmatrix}
  d_1 \\
  d_2 \\
  d_3 \\
  d_4 \\
  d_5 \\
  d_6 \\
\end{bmatrix} + (6 - \mathcal{C}) \begin{bmatrix}
  2 \\
  2 \\
  2 \\
  2 \\
  2 \\
  1 \\
\end{bmatrix}.
\]

All elements in the last vector are 2 except the last one, which is 1, because the dyad \{66\} has \( s_T = 2 \). For the rest of elements, the difference between the two ICVs is \( 2(6 - \mathcal{C}) \), which equals \( \mathcal{C}' - \mathcal{C} \). In the case of hexachords, \( 6 - \mathcal{C} = 0 \), so each hexachord and its complement have the same ICV. Therefore, if a hexachord class is not Z-related to any other one, then it is self-complementary. Conversely, if a hexachord class is Z-related to another one, then it is its complement (as shown at the end of section 4).

From (1), it follows that if set classes with the same cardinality are arranged with their ICVs in increasing or decreasing order, then so will the ICVs of their complements. This will make a set class and its complement to be in the same column in Table 1 (see section 6).

### 4. Trichord-Type and Trichord-Class Vectors

For \( \mathcal{C} = 3 \), there are 12 different set classes: the trichords, which will be arranged by decreasing ICV (or increasing prime IF). All of them have \( s_T = 1 \), except \{444\}, the augmented triad, which has \( s_T = 3 \). Five of them are inversionally symmetrical (including \{444\}). Thus, each of the remaining seven trichord classes is formed by two different trichord types, related by inversion. Both have the same ICV, but their sonorities are different, as well as their normal IFs. So, the one with lesser normal IF
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will be called “type a” and the other “type b”. For example, the trichord class \{345\} represents both the minor and major triads, whose normal IFs are, respectively, \{345\} and \{354\}, the former being type a and the latter type b. The total number of trichord types is, therefore, \(5 + 7 \times 2 = 19\).

For \(c > 3\), we can obtain the number of times each of the 19 trichord types is contained in a given set type. The result is a 19-element vector, which is here called the trichord-type vector (TTV). For clarity, the TTV will be written as two groups of numbers separated by a hyphen, the first one including the first 9 elements of the vector (corresponding to trichord types \{11A\}, \{129\}, \{192\}, \{138\}, \{183\}, \{147\}, \{174\}, \{156\}, and \{165\}, that is, those containing semitones) and the second one including the other 10 (corresponding to trichord types \{228\}, \{237\}, \{273\}, \{246\}, \{264\}, \{255\}, \{336\}, \{345\}, \{354\}, and \{444\}, that is, those not containing semitones). For a set type with cardinality \(c\), the sum of the elements of the TTV is \(\binom{c}{3} = c(c-1)(c-2)/6\). Appendix 2 contains the TTVs of all set classes and types, obtained with an original program written in MATLAB. It has been found that only 2 of them have the same TTV, which are the two types of set class \{112143\}. The rest of them have a unique TTV and therefore it fully characterizes their sonorities.

Let us consider a set class with two types. If the TTV of one type (a or b) is \(t_1 \cdots t_{19}\), then the TTV of the other type is obtained by simply interchanging the elements corresponding to non-inversionally-symmetrical trichords, as indicated in Figure 2. If a set class is inversionally symmetrical (only one type), the elements in the pairs shown in this figure are the same.

![Figure 2. Relation between the TTVs of types a and b of a set class.](image)

It is also possible to define a trichord-class vector (TCV) as the number of times each of the 12 trichord classes is contained in a given set class. The result is now a 12-element vector, whose elements are easily obtained from the TTV by simply adding the elements corresponding to non-inversionally-symmetrical trichords, as indicated in
Figure 3, where \( t_1 \cdots t_{19} \) are the elements of a TTV (either type, a or b) and \( f_1 \cdots f_{12} \) those of the TCV. Thus, the TTV is defined for the action of the cyclic group T, whereas the TCV is defined for the action of the dihedral group T/I. No two set classes have the same TCV, so the TCV fully characterizes a set class (this is true in \( \mathbb{Z}_{12} \), but for the corresponding study in \( \mathbb{Z}_n \) see the \( Z^3 \)-relation in Mandereau et al. 2011).

\[
\begin{array}{cccccccccccccc}
    t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 & t_9 & - & t_{10} & t_{11} & t_{12} & t_{13} & t_{14} & t_{15} & t_{16} & t_{17} & t_{18} & t_{19} \\
    f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 & f_9 & & f_{10} & f_{11} & f_{12} & & & & & & \\
\end{array}
\]

Figure 3. Relation between a TTV and the TCV of a set class.

Appendix 1 derives the relationship between the TTVs of a set type and its complement. Given a set type with cardinality \( c \), TTV \([t_1 \cdots t_{19}]\) and ICV \([d_1, \ldots, d_6]\), its complement will have the TTV \([t'_1 \cdots t'_{19}]\), which can be obtained by

\[
\begin{bmatrix}
    t'_1 \\
    t'_2 \\
    \vdots \\
    t'_{18} \\
    t'_{19}
\end{bmatrix} = T_h \begin{bmatrix}
    d_1 \\
    d_2 \\
    \vdots \\
    d_5 \\
    d_6
\end{bmatrix} - \begin{bmatrix}
    t_1 \\
    t_2 \\
    \vdots \\
    t_{18} \\
    t_{19}
\end{bmatrix} + (4 - c) \begin{bmatrix}
    3 \\
    3 \\
    \vdots \\
    3 \\
    1
\end{bmatrix},
\]  

(2)

where \( T_h \) is the matrix whose rows are the ICVs of the 19 trichord types, but multiplying the last column by 2 (the \( s_T \) of \{66\}) and dividing the last row by 3 (the \( s_T \) of \{444\}), and is given in (A13). All elements in the last vector are 3 except the last one, which is 1, because the trichord type \{444\} has \( s_T = 3 \).

On the other hand, the ICV and the TTV of a set type are related by

\[
\begin{bmatrix}
    d_1 \\
    d_2 \\
    d_3 \\
    d_4 \\
    d_5 \\
    d_6
\end{bmatrix} = \frac{1}{c-2} T' \begin{bmatrix}
    t_1 \\
    t_2 \\
    \vdots \\
    t_{18} \\
    t_{19}
\end{bmatrix}, \quad c \geq 3,
\]

(3)

where \( T' \) is the matrix whose columns are the ICVs of the 19 trichord types, which is the transpose of matrix T in (A10).
The TTVs of a set type and its complement are then related by

\[
\begin{bmatrix}
    t'_1 \\
    t'_2 \\
    t'_3 \\
    \vdots \\
    t'_{18} \\
    t'_{19}
\end{bmatrix} = \left( \frac{1}{c-2} T_h T' - I \right) \begin{bmatrix}
    t_1 \\
    t_2 \\
    t_3 \\
    \vdots \\
    t_{18} \\
    t_{19}
\end{bmatrix} + (4 - c) \begin{bmatrix}
    3 \\
    3 \\
    3 \\
    \vdots \\
    3 \\
    1
\end{bmatrix}, \quad c \geq 3, \quad (4)
\]

where \( I \) is the identity matrix of size 19, and the matrix product \( T_h T' \) is given in (A17).

For set classes not being inversionally symmetrical and with cardinality \( c > 3 \), the type with lexicographically greater TTV will be called type \( a \), and the other type \( b \). For example, the set class \( \{2334\} \) is formed by the types \( \{2334\} \) and \( \{2433\} \), corresponding to the half-diminished and dominant seventh chords, respectively, the former having greater TTV (as can be seen in Appendix 2), thus being type \( a \). This way, the complement of an \( a \)-type is always a \( b \)-type and vice versa. This can be seen directly from (2), where the greater \( [t_1 \cdots t_{19}] \) gives the lesser \( [t'_1 \cdots t'_{19}] \) and vice versa (the ICV \( [d_1, \ldots, d_6] \) is the same for both types of a set class). The only exception is the set class \( \{112143\} \), since its two types have the same TTV. Moreover, in this case, the complement of each type is itself; that is, they are self-complementary. So, in this case, the type \( a \) will be the one with lesser normal IF, that is, the prime IF. With these criteria, most prime IFs happen to be type \( a \).

For each pair of \( Z \)-related set classes, the member with greater TCV is here called \textit{hard} (because it turns out that it has the smaller intervals closer together) and the other \textit{soft}.

From (2), and taking into account Figure 3, it is obvious that the complement of a hard set class is a soft one and vice versa. Since this is also true for the hexachords, if a hexachord class is \( Z \)-related to another one, then it is its complement. For example, set classes \( \{12216\} \) and \( \{11235\} \) are \( Z \)-related, the latter having greater TCV, thus being hard, and the former soft. Their complements are, respectively, \( \{1111323\} \) (hard) and \( \{1112124\} \) (soft). These relations will be best visualized in Table 1, where these set classes correspond to 5-Z12, 5-Z36, 7-Z12, and 7-Z36 (see section 6).
5. List of Set Classes and Types

Usually, lists of set classes are given in one or several tables including the relevant information for each set class, such as: prime form (Forte or Rahn), Forte name, ICV and the degrees of transpositional and inversive symmetries (see, for example, Straus 2016). As well, every set class is placed across from its complement. Thus, on the one side (left) there are the trichords, tetrachords and pentachords, and on the other side (right) the nonachords, octachords, and heptachords. The hexachords form a separate group, where only those being Z-related are placed across from each other.

Forte names consist of two numbers separated by a hyphen, the first one corresponding to the cardinality and the second to an ordinal. For set classes with the same cardinality the ordinals are assigned by decreasing ICV, except in the case of Z-related pairs, where one member of each pair is placed at the end of the corresponding group. This way, every set class and its complement have the same ordinal. For example, the set class \{12414\} considered in section 2 is named 5-20, its complement being 7-20. Forte names of Z-related set classes include the letter “Z” just before the ordinal.

Appendix 2 is a detailed list of set classes and types. They are first grouped by cardinality and then arranged by decreasing ICV, with the ties being arranged by decreasing TCV (the hard set class in a Z-related pair before the soft one) and further ties by decreasing TTV (the a-type before the b-type). The only remaining tie (the two types of set class \{112143\} or 6-14) is arranged by increasing normal IF. Complementary set classes are placed “next to” each other (vertically in the case of hexachords and horizontally in the other cases). To simplify the notation, no brackets or braces are used in the list. For each set class or type, the information is given in 6 columns containing the following:

1) A general ordinal, ranging from 0 to 351. Set classes being inversionally symmetrical \(s_i = s_T\) are indicated by a hyphen just after the ordinal (no hyphen means \(s_i = 0\)). Regarding the degree of transpositional symmetry, when \(s_T > 1\) it is given as a superscript after the ordinal or the hyphen (no superscript means \(s_T = 1\)).
2) The normal IF. For most non-inversionally-symmetrical set classes ($s_i = 0$), their prime IFs are type a, but when they are type b an asterisk (*) is included just after the normal IF.

3) The Rahn normal form. For most non-inversionally-symmetrical set classes, their Rahn prime forms are type a, but when they are type b it is indicated by a superscript with a plus sign (+). As well, most Forte normal forms are equal to Rahn’s, but when different a superscript with a letter is included, which refers to the Forte normal form at the end of the list (none of them turn out to have the plus sign).

4) An extended Forte name, including a letter (“a” or “b”) to indicate the type, when applicable. As well, Z-related set classes include a superscript with the ordinal of the other member of the pair. When looking for the complement of a set type, remember that the complement of an a-type is a b-type and vice versa, except for the two types of set class \{112143\} (6-14a and 6-14b), which are self-complementary. This special case is indicated by a superscript with an equal sign (=), representing that both types have the same TTV and type a corresponds to the prime IF.

5) The ICV.

6) The TTV. The inclusion of this characteristic is the most significant difference with respect to previously published lists of set classes.

**6. Periodic Table of Set Classes**

It would be interesting to have a compact version of the list of set classes and types, where the main information and the relationship among them can be seen at a glance. Table 1 serves this purpose. As will be explained below, it contains all set classes represented by their Forte names, together with their degrees of transpositional and inversional symmetries, and some other details. For every set class, its complement is easily found, as well as its Z-related set class, if it has one. The prime form chosen for characterizing the set classes is the IF, since it has proved to be more versatile than the others and, additionally, some characteristics of the table are based on it. To simplify the notation, the prime IFs are written without braces. Moreover, when a prime IF contains several semitones in a row they are represented as a power of 1 (for example, 111 is represented as $1^3$). Table 1 has been developed in the following way:
1) Set classes are first grouped by cardinality and each of these groups is called a *period*. Periods are arranged by increasing cardinality and are represented by the cardinality followed by a hyphen (left column in Table 1), as in the initial part of Forte names. To make the table more compact, periods 0, 1, and 2 are placed in a single row, as are periods 12, 11, and 10.

2) Each period starts with the set class whose pitch classes are the closest together (that is, in a chromatic sequence) and ends with the set class whose pitch classes are the most evenly spaced. Thus, for example, period 4 starts with \{1119\} and ends with \{3333\}.

3) Within a period, set classes are arranged by decreasing ICV and are assigned the same ordinals as in Forte names (the big numbers in the cells of Table 1). This way, the number of smaller intervals in the set classes decreases within a period, which matches the previous criterion; and, additionally, each set class and its complement are placed in the same column. Z-related set classes share a cell, so that they are easily identified and each cell has a unique ICV (for \(c \geq 2\)). For each Z-related pair, the member with greater TCV (hard) is placed in the upper part of the cell. Remember that the complement of a hard set class is a soft one and vice versa. For example, the complement of 5-Z12 (soft) is 7-Z12 (hard). And, since 5-36 is not inversionally symmetrical, the complements of 5-Z36a and 5-Z36b (hard) are, respectively, 7-Z36b and 7-Z36a (soft).

4) Set classes being inversionally symmetrical (\(s_1 = s_\tau\)) have the ordinal underlined. Regarding the degree of transpositional symmetry, when \(s_\tau > 1\) it is given as a superscript on the ordinal (no superscript means \(s_\tau = 1\)).

5) For set classes not being inversionally symmetrical (\(s_1 = 0\)), those whose prime IFs are type b have an asterisk on the ordinal, and those whose Rahn (and Forte) prime forms are type b have a superscript with a plus sign (+). Otherwise, they are type a. In the special case of 6-14, a superscript with the equal sign (=) is used to indicate that its two types, 6-14a and 6-14b, are the only ones with the same TTV and the type a corresponds to the prime IF (but the Rahn and Forte prime forms are type b, so a plus sign is included, too). Additionally, both are self-complementary.

6) The prime IFs are given just below the ordinals. Furthermore, in order to facilitate finding a prime IF in its corresponding period, set classes with the same number of semitones (number of 1’s in the prime IF or first element in the ICV) are assigned
the same cell colour (white or grey in the printed version of the journal). And that colour is also assigned to their complements.

To sum up, the periodic table provides the following information: the Forte names, the degrees of transpositional and inversional symmetries, the Z and complement relations, the prime IFs and the types of all prime forms. Additionally, Forte and Rahn normal forms are easily obtained from the prime IFs and their corresponding types. Moreover, given the prime IF of a set class, it is easy to find it in the table.

Table 2 shows, for each period, the number of set classes having the same number of semitones, that is, with the same cell colour in Table 1 (a similar arrangement is performed in both tables). As well, it shows, for each period, the number of set classes, set types and pitch-class sets. The special case of set class 0-1 (null set) is also included, so the total number of pitch-class sets is, of course, $2^{12} = 4096$. Note the symmetries between complementary periods.
A Detailed List and a Periodic Table of Set Classes

Table 1. Periodic table of set classes.
Table 2. Number of set classes, set types and pitch-class sets.

<table>
<thead>
<tr>
<th>Period</th>
<th>12</th>
<th>11</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1-</td>
<td>1</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-</td>
<td></td>
<td>1</td>
<td>5</td>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-</td>
<td></td>
<td>1</td>
<td>4</td>
<td>7</td>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-</td>
<td></td>
<td>1</td>
<td>8</td>
<td>12</td>
<td>8</td>
<td></td>
<td>29</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-</td>
<td></td>
<td>1</td>
<td>6</td>
<td>18</td>
<td>10</td>
<td>3</td>
<td>38</td>
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<tr>
<td>6-</td>
<td></td>
<td>1</td>
<td>9</td>
<td>19</td>
<td>17</td>
<td>3</td>
<td>50</td>
<td></td>
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<tr>
<td>7-</td>
<td></td>
<td>1</td>
<td>6</td>
<td>18</td>
<td>10</td>
<td>3</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>8-</td>
<td></td>
<td>1</td>
<td>8</td>
<td>12</td>
<td>8</td>
<td></td>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9-</td>
<td></td>
<td>1</td>
<td>4</td>
<td>7</td>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-</td>
<td></td>
<td>1</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11-</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12-</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>5</td>
<td>16</td>
<td>19</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>47</td>
<td>30</td>
<td>26</td>
</tr>
</tbody>
</table>

7. Conclusions

In this article, I represent each set type by the intervallic form, while the trichord-type vector fully characterizes its sonority, with only one exception. As well, I derive the formulas relating the trichord-type vectors of a set type and its complement. When dealing with set classes instead of set types, we can use the trichord-class vector, which is easily obtained from the trichord-type vector and, unlike the interval-class vector, fully characterizes a set class. Then, I provide a detailed list of set classes and types, including for each one the normal intervallic form, both Rahn and Forte normal forms, an extended Forte name, the interval-class vector and the trichord-type vector. The inclusion of this last characteristic represents a relevant contribution compared to the usual lists found in the literature. Finally, I develop a compact periodic table containing all set classes, which shows their main characteristics and relationships at a glance. The interval-class, trichord-class, and trichord-type vectors played a crucial role in arranging the set classes and types, both in the detailed list and the periodic table.

8. Acknowledgments

I am grateful to the Editor Jason Yust and the anonymous reviewers for their valuable comments on the manuscript, which contributed to improve its quality.
9. References


A Detailed List and a Periodic Table of Set Classes


Appendix 1: Formulas for the Interval-Class and Trichord-Type Vectors

This Appendix gives the derivation of the formulas included in sections 3 and 4 relating the ICVs and TTVs of a set class or set type and its complement. Now, another representation of a pitch-class set, considered as a subset of $\mathbb{Z}_{12}$, will be used: the characteristic function, which will be written in brackets without commas. For example, the pitch-class set [95A24], considered in section 2, has the characteristic function [00101100110], as it contains the pitch classes 2, 4, 5, 9 and A. The characteristic function of its complement is obtained by simply substituting every 1 with a 0 and vice versa, that is, [11010011101]. When dealing with a set class or set type, the characteristic function of any of their pitch-class sets can be used. Lewin (1960) provides the first formula in this respect, corresponding to the so-called “intervallic content,” which is similar to the ICV. The ICV formula can be derived as follows:

Let $S$ and $S'$ be the characteristic functions of a given set class and its complement. Their circular autocorrelations modulo 12 are, respectively,

$$P(n) = \sum_{k=0}^{11} S(k)S(k + n), \quad P'(n) = \sum_{k=0}^{11} S'(k)S'(k + n), \quad n = 0, ..., 11,$$  \hspace{1cm} (A1)

where $k + n$ is understood modulo 12. Note that $P(n) = P(12 - n)$.

As $S'(k) = 1 - S(k),$

$$P'(n) = \sum_{k=0}^{11} [1 - S(k)][1 - S(k + n)] = 12 - 2c + P(n),$$  \hspace{1cm} (A2)

$c$ being the cardinality of the set class $S$.

The ICV $[d_1 \cdots d_6]$ of $S$ is then

$$d_n = \frac{P(n)}{s_n}, \quad n = 1, ..., 6,$$  \hspace{1cm} (A3)

where $s_n$ is the degree of transpositional symmetry (written as $s_T$ in section 2) of the $n$-th dyad, which is $s_n = 1$ for $n = 1, ..., 5$ and $s_6 = 2$, the latter corresponding to the tritone. Due to its symmetry, each tritone in $S$ adds 2 to $P(6)$, so it is necessary to divide it by $s_6 = 2$ to obtain the correct value of $d_6$. Thus, formulas (A2) and (A3) give the vector equation.
A Detailed List and a Periodic Table of Set Classes

\[
\begin{bmatrix}
  d_1' \\
  d_2' \\
  d_3' \\
  d_4' \\
  d_5' \\
  d_6'
\end{bmatrix} =
\begin{bmatrix}
  d_1 \\
  d_2 \\
  d_3 \\
  d_4 \\
  d_5 \\
  d_6
\end{bmatrix} + (6 - c) \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 1 \end{bmatrix}. \tag{A4}
\]

Therefore, given the ICV of a set class, equation (A4) gives the ICV of its complement.

For the TTV formula, I generalize the previous procedure by defining the functions

\[
Q(i, j) = \sum_{k=0}^{11} S(k)S(k + i)S(k + j), \quad i, j = 0, \ldots, 11, \tag{A5}
\]

\[
Q'(i, j) = \sum_{k=0}^{11} S'(k)S'(k + i)S'(k + j)
\]

where \(k + i\) and \(k + j\) are understood modulo 12.

As \(S'(k) = 1 - S(k)\),

\[
Q'(i, j) = \sum_{k=0}^{11} [1 - S(k)][1 - S(k + i)][1 - S(k + j)], \tag{A6}
\]

which, taking into account that

\[
\sum_{k=0}^{11} S(k + i)S(k + j) = P(j - i), \tag{A7}
\]

gives

\[
Q'(i, j) = 12 - 3c + P(i) + P(j) + P(j - i) - Q(i, j), \tag{A8}
\]

c being the cardinality of the set type \(S\). To calculate the elements of the TTV, the functions \(Q(i, j)\) and \(Q'(i, j)\) are written as \(Q(n)\) and \(Q'(n)\) following the relations in Table A.1 (obtained from the Rahn or Forte normal forms of the 19 trichord types, excluding the first 0).

Table A.1. Relation between \(n\) and the pair \(i, j\) for \(n = 1, \ldots, 19\)

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i, j)</td>
<td>1, 2</td>
<td>1, 3</td>
<td>2, 3</td>
<td>1, 4</td>
<td>3, 4</td>
<td>1, 5</td>
<td>4, 5</td>
<td>1, 6</td>
<td>5, 6</td>
</tr>
<tr>
<td>(n)</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>(i, j)</td>
<td>2, 4</td>
<td>2, 5</td>
<td>3, 5</td>
<td>2, 6</td>
<td>4, 6</td>
<td>2, 7</td>
<td>3, 6</td>
<td>3, 7</td>
<td>4, 7</td>
</tr>
</tbody>
</table>
The TTV \([t_1 \cdots t_{19}]\) of \(S\) is then

\[
t_n = \frac{Q(n)}{r_n}, \quad n = 1, \ldots, 19,
\]

where \(r_n\) is the degree of transpositional symmetry (written as \(s_T\) in section 2) of the \(n\)-th trichord type, which is \(r_n = 1\) for \(n = 1, \ldots, 18\) and \(r_{19} = 3\), the latter corresponding to the augmented triad. Due to its symmetry, each augmented triad in \(S\) adds 3 to \(Q(19)\), so it is necessary to divide it by \(r_{19} = 3\) to obtain the correct value of \(t_{19}\).

Now, let \(T\) be the matrix whose rows are the ICVs of the 19 trichord types, which is

\[
T = \begin{bmatrix}
2 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 \\
0 & 2 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 2 & 0 \\
0 & 0 & 2 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 3 & 0 & 0
\end{bmatrix}
\]  

Then, each trichord type \(n\) contains 3 dyads that are given by the corresponding row of matrix \(T\). And, taking into account the relation between \(n\) and the pair \(i, j\) in Table A.1, the functions \(P(i), P(j),\) and \(P(j - i)\) in (A8) are precisely the number of times each of those dyads is contained in \(S\) (except the tritones, which are recorded twice). On the other hand, the actual dyads in \(S\) are given by its ICV \((d_k, k = 1, \ldots, 6)\). Therefore, for each trichord type \(n\),

\[
P(i) + P(j) + P(j - i) = \sum_{k=1}^{6} T_{nk} d_k s_k, \quad n = 1, \ldots, 19
\]

Thus, formulas (A8), (A9), and (A11) give the matrix equation.
A Detailed List and a Periodic Table of Set Classes

\[
\begin{bmatrix}
 t'_1 \\
 t'_2 \\
 t'_3 \\
 \vdots \\
 t'_{19}
\end{bmatrix}
= T_h
\begin{bmatrix}
 d_1 \\
 d_2 \\
 d_3 \\
 \vdots \\
 d_6
\end{bmatrix}
= \begin{bmatrix}
 t_1 \\
 t_2 \\
 t_3 \\
 \vdots \\
 t_{18}
\end{bmatrix} + (4 - c) \begin{bmatrix}
 3 \\
 3 \\
 3 \\
 \vdots \\
 1
\end{bmatrix}. \tag{A12}
\]

\( T_h \) being the matrix \( T \), but by multiplying its last column by \( s_6 = 2 \) and dividing its last row by \( r_{19} = 3 \), that is,

\[
T_h = \begin{bmatrix}
 2 & 1 & 0 & 0 & 0 & 0 \\
 1 & 1 & 1 & 0 & 0 & 0 \\
 1 & 1 & 1 & 0 & 0 & 0 \\
 1 & 0 & 1 & 1 & 0 & 0 \\
 1 & 0 & 1 & 1 & 0 & 0 \\
 1 & 0 & 0 & 1 & 1 & 0 \\
 1 & 0 & 0 & 0 & 1 & 2 \\
 1 & 0 & 0 & 0 & 1 & 2 \\
 0 & 2 & 0 & 1 & 0 & 0 \\
 0 & 1 & 1 & 0 & 1 & 0 \\
 0 & 1 & 1 & 0 & 1 & 0 \\
 0 & 1 & 0 & 1 & 0 & 2 \\
 0 & 1 & 0 & 1 & 0 & 2 \\
 0 & 1 & 0 & 0 & 2 & 0 \\
 0 & 0 & 2 & 0 & 0 & 2 \\
 0 & 0 & 1 & 1 & 1 & 0 \\
 0 & 0 & 1 & 1 & 1 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}. \tag{A13}
\]

Therefore, given the TTV and the ICV of a set type, equation (A12) gives the TTV of its complement.

Additionally, it is possible to obtain the ICV of a set type \( S \) from its TTV. Each column \( k \) in matrix \( T \) gives the number of times the corresponding dyad is contained in the different trichord types. And the actual trichord types in \( S \) are given by its TTV \( (t_n, n = 1, \ldots, 19) \). On the other hand, if the cardinality of \( S \) is \( c \geq 3 \), each dyad in \( S \) forms a trichord type with each of the other pitch classes in \( S \), so that dyad is contained a total of \( c - 2 \) times in the trichord types in \( S \). Therefore, for each dyad \( k \),

\[
\sum_{n=1}^{19} T_{nk}t_n = (c - 2)d_k, \quad k = 1, \ldots, 6, \quad c \geq 3, \tag{A14}
\]

which can be written in matrix form as
A Detailed List and a Periodic Table of Set Classes

\[
\begin{bmatrix}
    d_1 \\
    d_2 \\
    d_3 \\
    d_4 \\
    d_5 \\
    d_6
\end{bmatrix} = \frac{1}{c-2} T' \begin{bmatrix}
    t_1 \\
    t_2 \\
    t_3 \\
    \vdots \\
    t_{18} \\
    t_{19}
\end{bmatrix}, \quad c \geq 3, \quad (A15)
\]

\(T'\) being the transpose of matrix \(T\) in (A10).

Thus, from (A12) and (A15),

\[
\begin{bmatrix}
    t_1' \\
    t_2' \\
    t_3' \\
    \vdots \\
    t_{18}' \\
    t_{19}'
\end{bmatrix} = \left(\frac{1}{c-2} T_h T' - I\right) \begin{bmatrix}
    t_1 \\
    t_2 \\
    t_3 \\
    \vdots \\
    t_{18} \\
    t_{19}
\end{bmatrix} + (4 - c) \begin{bmatrix}
    3 \\
    3 \\
    3 \\
    \vdots \\
    3 \\
    1
\end{bmatrix}, \quad c \geq 3, \quad (A16)
\]

where \(I\) is the identity matrix of size 19, and the matrix product \(T_h T'\) is

\[
T_h T' =
\begin{bmatrix}
    5 & 3 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
    3 & 3 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 1 & 0 \\
    3 & 3 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 1 & 1 & 0 \\
    2 & 2 & 2 & 3 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 3 \\
    2 & 2 & 2 & 3 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 3 \\
    2 & 1 & 1 & 2 & 2 & 3 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 3 \\
    2 & 1 & 1 & 2 & 2 & 3 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 3 \\
    2 & 1 & 1 & 1 & 1 & 2 & 2 & 4 & 4 & 0 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 & 0 \\
    2 & 1 & 1 & 1 & 1 & 2 & 2 & 4 & 4 & 0 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 & 0 \\
    2 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 5 & 2 & 2 & 3 & 3 & 2 & 0 & 1 & 1 & 3 \\
    1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 3 & 3 & 1 & 1 & 3 & 2 & 2 & 2 & 0 \\
    1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 3 & 3 & 1 & 1 & 3 & 2 & 2 & 2 & 0 \\
    1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 3 & 3 & 1 & 1 & 4 & 4 & 1 & 2 & 1 & 1 & 3 \\
    1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 3 & 3 & 1 & 1 & 4 & 4 & 1 & 2 & 1 & 1 & 3 \\
    1 & 1 & 1 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 3 & 3 & 1 & 1 & 5 & 0 & 2 & 2 & 0 \\
    0 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 0 & 0 & 6 & 2 & 2 & 0 \\
    0 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 2 & 3 & 3 & 3 \\
    0 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 2 & 3 & 3 & 3 \\
    0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 3
\end{bmatrix}
\]

Therefore, given the TTV of a set type, equation (A16) gives the TTV of its complement.
## Appendix 2: List of Set Classes and Types

Columns: 1) General ordinal - 2) Normal IF - 3) Rahn normal form - 4) Extended Forte name - 5) ICV - 6) TTV

<table>
<thead>
<tr>
<th>c = 0</th>
<th>0-12</th>
<th>000000 00000000-0000000000</th>
<th>351-353</th>
<th>0123456789AB 12-1</th>
<th>CCCCC6</th>
<th>CCCCCCCC-CCCCCCCCC4</th>
</tr>
</thead>
<tbody>
<tr>
<td>c = 1</td>
<td>1-</td>
<td>0 1-1 000000 00000000-0000000000</td>
<td>350-351</td>
<td>0123456789A 11-1</td>
<td>AAAA5</td>
<td>999999999-9999999993</td>
</tr>
<tr>
<td>c = 2</td>
<td>2-</td>
<td>1B 01 2-1 100000 00000000-0000000000</td>
<td>344-345</td>
<td>0123456789A 12-1</td>
<td>988884</td>
<td>877777777-6666666662</td>
</tr>
<tr>
<td>c = 3</td>
<td>3-</td>
<td>11A 02 3-1 210000 00000000-0000000000</td>
<td>325-326</td>
<td>0123456789A 12-1</td>
<td>976666</td>
<td>655544444445544544</td>
</tr>
<tr>
<td>c = 4</td>
<td>4-</td>
<td>12 03 4-1 321000 21100000-000000000000</td>
<td>327-328</td>
<td>0123456789A 12-1</td>
<td>976666</td>
<td>655544444445544544</td>
</tr>
<tr>
<td>c = 5</td>
<td>5-</td>
<td>13 04 5-1 432000 32100000-000000000000</td>
<td>329-330</td>
<td>0123456789A 12-1</td>
<td>976666</td>
<td>655544444445544544</td>
</tr>
<tr>
<td>c = 6</td>
<td>6-</td>
<td>14 05 6-1 543000 43200000-000000000000</td>
<td>331-332</td>
<td>0123456789A 12-1</td>
<td>976666</td>
<td>655544444445544544</td>
</tr>
<tr>
<td>c = 7</td>
<td>7-</td>
<td>15 06 7-1 654000 54300000-000000000000</td>
<td>333-334</td>
<td>0123456789A 12-1</td>
<td>976666</td>
<td>655544444445544544</td>
</tr>
<tr>
<td>c = 8</td>
<td>8-</td>
<td>16 07 8-1 765000 65400000-000000000000</td>
<td>335-336</td>
<td>0123456789A 12-1</td>
<td>976666</td>
<td>655544444445544544</td>
</tr>
<tr>
<td>c = 9</td>
<td>9-</td>
<td>17 08 9-1 876000 76500000-000000000000</td>
<td>337-338</td>
<td>0123456789A 12-1</td>
<td>976666</td>
<td>655544444445544544</td>
</tr>
<tr>
<td>c = 10</td>
<td>10-</td>
<td>18 09 10-1 987000 87600000-000000000000</td>
<td>339-340</td>
<td>0123456789A 12-1</td>
<td>976666</td>
<td>655544444445544544</td>
</tr>
<tr>
<td>c = 11</td>
<td>11-</td>
<td>19 10 11-1 109000 98700000-000000000000</td>
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A Detailed List and a Periodic Table of Set Classes
A Detailed List and a Periodic Table of Set Classes
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Forte normal forms that are different from Rahn's:

- 27/27 -

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