TENOR

INTERNATIONAL CONFERENCE ON TECHNOLOGIES FOR MUSIC NOTATION & REPRESENTATION

Hamburg University for Music and Theater (HfMT)
Hamburg, Germany

Sixth International Conference on Technologies for Music Notation and Representation
Online Edition

Edited by:
Rama Gottfried, Georg Hajdu, Jacob Sello,
Alessandro Anatrini, and John MacCallum.

ISBN 978-3-00-066930-9

International Conference on Technologies for Music Notation and Representation TENOR 2020
Hamburg, Germany (Online Edition)

Proceedings published by: Hamburg University for Music and Theater

Editorial coordination: Rama Gottfried

Logo design: Nicolas Taffin, updated for 2020 by Janina Luckow

REPRESENTATION OF HARMONIES ON THE HARMONIC WHEEL

Luis Nuño

ITACA Institute
Polytechnic University of Valencia (Spain)
lnuno@dcom.upv.es

ABSTRACT

The Tonnetz is a useful tool for representing musical excerpts or full pieces containing mainly major and minor triads. However, when a musical composition contains dissonant triads or higher-order chords, it can only give a limited representation of it. The Harmonic Wheel is a physical tool that combines a Tonnetz transformed into a polar grid with a plastic disc containing the lines that define the major, harmonic and melodic minor scales, together with the scale degrees and the symbols of the corresponding seventh chords. This way, it allows to represent a large variety of musical works, including both triads and seventh chords, as well as to find the chords that are common to different keys. To show its main characteristics and advantages, several examples are given from different musical styles. In all cases, the representations obtained are simple and compact, and therefore easy to memorize, which makes the Harmonic Wheel a powerful and versatile tool for analyzing and composing music, as well as providing an efficient mnemonic notation.

1. INTRODUCTION

The Tonnetz is a graphic representation of musical notes and their consonance relationships, that is, the consonant intervals (perfect fifth, major and minor thirds) and the consonant triads (major and minor) formed by them. There are two relevant representations of it: the Oettingen/Riemann and the Douthett and Steinbach, which are dual [1, 2]. The Oettingen/Riemann Tonnetz (Figure 1) is a triangular lattice, where the notes are at the vertices and the consonant triads on the triangles, while the Douthett and Steinbach's Tonnetz (also called Chicken-Wire) is a hexagonal lattice, where the notes are on the hexagons and the consonant triads at the vertices. In both cases, the edges represent the consonant intervals, which in turn define (in different ways) the P, L and R operations, which stand for parallel, leading-tone exchange and relative, respectively.

They are defined for consonant triads and, for example, P maps C major to C minor, L maps C major to E minor and R maps C major to A minor, and vice versa [1].

Both Tonnetze are infinite in a plane, that is, in a 2-dimensional (2D) space. However, they are periodic in 3 directions, so, by choosing any 2 of them, we can obtain an alternative representation on a torus, which is a finite surface in a 3-dimensional (3D) space.

Generalizations of the Tonnetz to include other (dissonant) intervals and trichords or higher-order chords, such as tetra- or pentachords, lead to more complex geometries, which require 3 or more space dimensions [3 – 6].

Both the analysis and composition of some kinds of musical pieces can be greatly simplified by representing their notes and harmonies on a Tonnetz, mainly when they only include major and minor triads. Logically, 2D graphs are simpler and easier to use than those requiring 3 or more space dimensions.

In this respect, some examples of representing musical excerpts on a 2D Tonnetz are given in [7], which only contain major and minor triads. They consist of binary and ternary combinations of PLR operations and correspond to nineteenth century music by Brahms, Schubert, Beethoven, Verdi and Wagner. Other examples, from mid-twentieth century Jazz and Latin repertoire, are given in [8], where most harmonies are seventh chords. Therefore, in order to represent them on a 2D Tonnetz, the seventh was omitted in the dominant seventh chords and the root was omitted in the half-diminished chords.

The Harmonic Wheel [9] is a practical 2D Tonnetz, where one of the axes is re-oriented so that the notes of a major key form a rectangle, thus resulting in a full rectangular grid, which is then transformed into a polar one. The final graph is an annulus, which is finite in a plane, thus keeping the advantages of both the planar and the toroidal Tonnetze. Additionally, the regions corresponding to the major, harmonic and melodic minor scales, together with the scale degrees and the seventh chords associated to them, are also indicated, which facilitates the representation of harmonies of tonal pieces.

To show its main characteristics and advantages, first a couple of examples on diatonic modulation are given, which deal with finding the pivot chords. Then, the harmonies of five musical excerpts and pieces are represented on the Harmonic Wheel: an excerpt by Beethoven included in [7], a well-known tonal song, a Coltrane's composition included in [8] (but represented there on a chromatic circle instead of a Tonnetz), a piece whose harmonies are based on Béla Bartók's axes, and a song including modulations.

In all cases, the representations obtained are simple and compact, and therefore easy to memorize, which makes this representation system an efficient mnemonic notation. It is worth pointing out that some of the examples here presented are, in some cases, studied in the 12 keys, so having such a mnemonic notation is greatly helpful. In fact, the Harmonic Wheel, together with other similar tools, are part of a subject of a Master on Music and Scenic Arts in a Doctorate Program.

2. HARMONIC WHEEL

Figure 1 shows the Oettingen/Riemann Tonnetz, where the notes are assigned to the vertices and the major and minor triads to the triangles. The horizontal lines represent the perfect fifths and the two oblique lines the major and minor thirds. Additionally, a region containing the 12 major and 12 minor triads just once is marked with a dashed line.

In Figure 2, the notes in that region belonging to the C major key are marked with circles, except the tonic, which is marked with a rectangle. Then, the two oblique sides are re-oriented to become vertical (Figure 3), so that the notes of a major key form a rectangle (the 3 notes of the C major key outside the rectangle are in fact repeated on it) and, therefore, the whole grid becomes rectangular. Furthermore, the 6 consonant triads belonging to the C major key are inside the rectangle and the C major and A minor triads are in its centre. As well, each pair of relative triads forms a smaller rectangle, which is assigned the corresponding key signature. This way, each triad also represents the centre of a major or natural minor key.

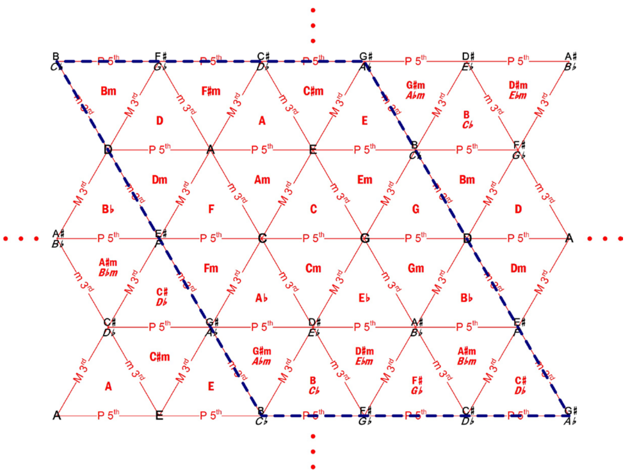

Figure 1. Oettingen/Riemann Tonnetz and a region with the 12 major and 12 minor triads.

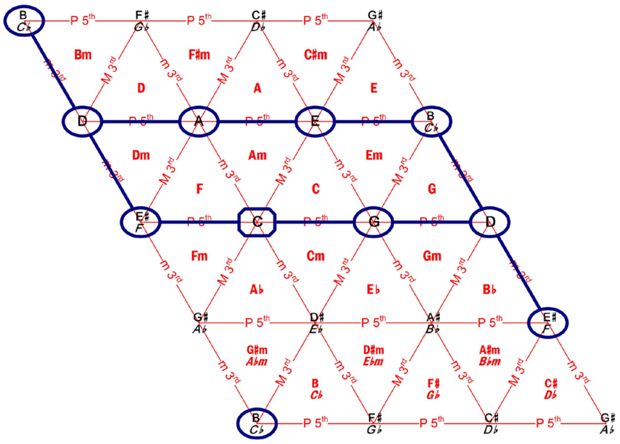

Figure 2. The C Major key on the Oettingen/Riemann Tonnetz.

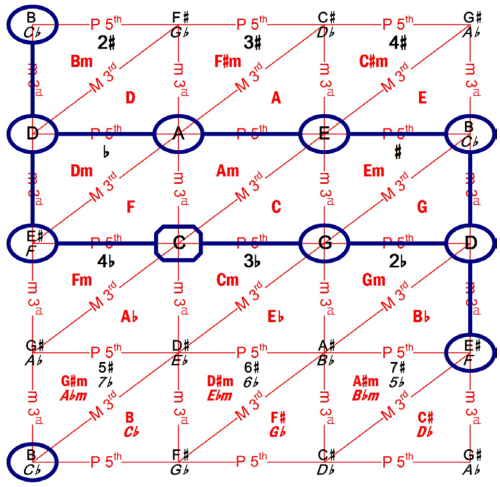

Figure 3. The C Major key on a rectangular grid.

In Figure 3, the notes at the bottom are repeated at the top, which means that this diagram is cyclic in the vertical direction. In contrast, obtaining a cycle in the horizontal direction requires to add more notes and triads until completing a cycle of fifths.

If we do so and then curve the diagram to make it circular, the result is the Harmonic Wheel, shown in Figure 4. This way, the rectangular grid is transformed into a polar one, so that the horizontal lines turn into circumferences, the vertical lines into radii and the diagonal lines into spirals. Consequently, the 3 types of cycles are now as follows: a closed cycle on each circumference, containing 12 perfect fifths (or fourths); an open cycle along each radius, containing 4 minor thirds; and an open cycle on each spiral, containing 3 major thirds. The first two cycles are clearly seen on the graph, while the last one is not so evident, and this is the reason why it was chosen with the least number of intervals (3).

In practice, the Harmonic Wheel is a physical tool consisting of two rotating discs: one cardboard, with the full polar grid printed on it (including the notes, interval lines, triads and key signatures, in black and red colours), and the other a transparent plastic, with the lines defining a major key printed on it (in blue). The two discs are joined together at their centres with a rivet, which allows selecting any major or natural minor key, together with its corresponding major and minor triads. To compare this tool with a 3D Tonnetz, Figure 5 shows a handmade toroidal Tonnetz, where the blue lines correspond to a major key (in this case, E♭ major). The difficulties for using it in practice are apparent.

Returning to the Harmonic Wheel, the scale degrees and the seventh-chord symbols are also printed on the plastic disc in blue (Figure 6). And, because the major scale region only takes up one fourth of that disc, in the final design two other fourths are utilized to print the corresponding lines, scale degrees and seventh-chord symbols for the harmonic and melodic minor scales, while the other fourth remains free.

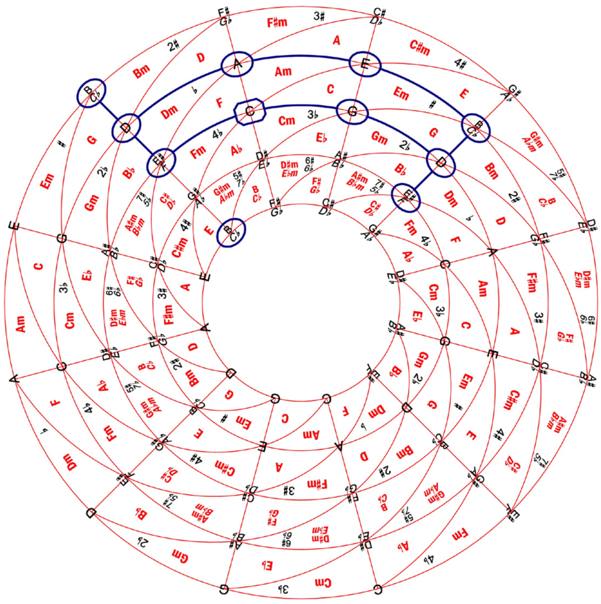

Figure 4. Harmonic Wheel and the C Major key (polar grid).

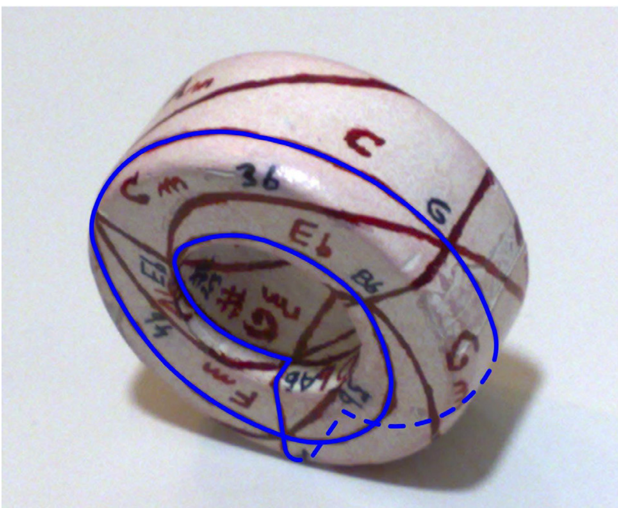

Figure 5. Handmade toroidal Tonnetz and the E♭ Major key.

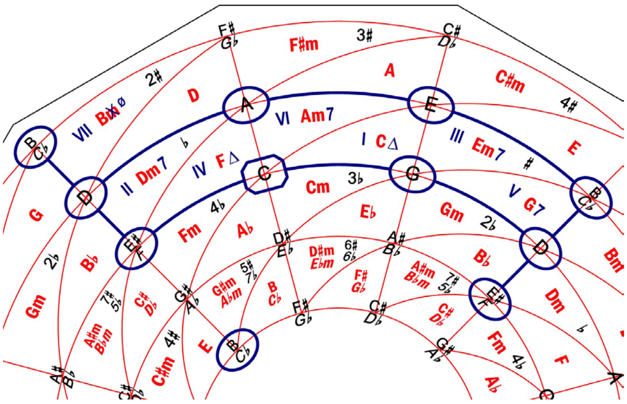

Figure 6. Scale degrees and seventh chords of C Major on the Harmonic Wheel.

Logically, the triads associated to each degree of those scale types are obtained by simply omitting the sevenths in the seventh chords. This includes not only major and minor triads, but also augmented and diminished.

The addition of the plastic disc to the polar grid, which allows selecting any major or minor scale (natural, harmonic or melodic) with its corresponding scale degrees, triads and seventh chords, makes this tool a powerful and versatile resource for analyzing and composing a variety of musical styles. To show it, some examples are given in section 4, where, for simplicity, only the bare diagram and the major scale will be used.

3. INTERVALS AND CHORDS

The red lines in both the rectangular and polar grids represent the consonant intervals, that is, the perfect fifths, the major and the minor thirds (as well as their inversions). The rest of intervals, which are dissonant, are the semitone, the tone and the tritone. Every interval belongs to an "interval class" characterized by its minimum number of semitones (considering the given interval and its inversion within an octave), so there are 6 interval classes, named 2-1 to 2-6 after Forte [10]. Here, we will represent them by icn, n being the number of semitones, which ranges from 1 to 6. Interval classes $ic3$, $ic4$ and $ic5$ are directly represented on the grids, whereas $ic1$, $ic2$ and $ic6$ can be expressed as a combination of two of the previous ones by

$$ic1 = ic4 - ic3 = ic5 - ic4 \quad (1)$$

$$ic2 = ic5 - ic3 = -2 \cdot ic5 \quad (2)$$

$$ic6 = 2 \cdot ic3 \quad (3)$$

Additionally, the following equation holds:

$$ic3 + ic4 + ic5 = 0 \quad (4)$$

which means that the sum of the 3 consonant intervals gives a closed line, which in turn defines a triangular area corresponding to a consonant triad, major or minor. All these equations have been represented graphically in Figure 7, where, for simplicity, a rectangular grid was considered. Of course, the corresponding representations on a polar grid are easy to visualize mentally.

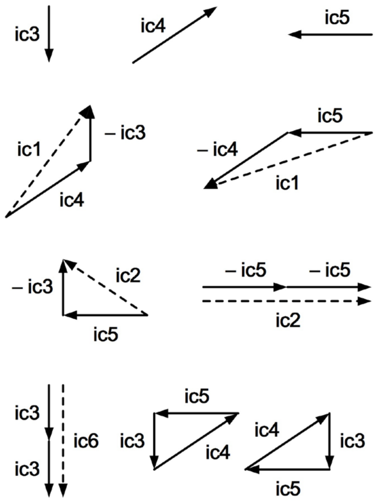

Figure 7. Interval classes on a rectangular grid.

In pitch-class set theory, every "set class" (a generalization of chord type) is assigned a Forte name consisting of two numbers separated by a hyphen, the first one corresponding to the "cardinality" (the number of "pitch-classes" or notes in the set class) and the second to an ordinal. For example, a diminished triad (3 pitch-classes) is named 3-10, and a minor seventh chord (4 pitch-classes) 4-26.

A practical way to describe the structure of a "pitch-class set" (a particular chord) is by means of the "intervallic form" [11], which is the sequence of intervals (in semitones) between every two adjacent pitch classes, including the interval between the last and the first ones. For example, the intervallic form of a major chord, such as C major, is {435}, because the intervals between its adjacent notes (C E G) are 4, 3, and 5 semitones (the latter being the interval from G to C). The circular shifts of this intervallic form, which are {354} and {543}, also correspond to the same chord (but starting from a different note). For a minor chord, the intervallic form is {345} or any of its circular shifts. As it is equal to the intervallic form of a major chord, but in reverse order, a minor chord is said to be the "inversion" of a major chord, and they two form a set class, whose Forte name is 3-11. So, in order to distinguish between them, a letter "a" or "b" can be added to the Forte name (a general criterion for assigning them is provided in [11]). As a last example, the intervallic form of a diminished triad (set class 3-10) is {336}. Since it is "inversionally symmetrical", no letter will be assigned to it.

To relate chords to intervals, for set classes with 3 or more pitch-classes (as trichords and tetrachords), an "interval-class vector" is defined, which has 6 components that list the number of times each interval class ($ic1$ to $ic6$) is contained in the given set class. For example, the interval-class vector of a diminished triad is (002001), because it contains 2 interval-classes $ic3$ and one $ic6$, and the interval-class vector of an augmented triad is (000300), as it contains 3 interval-classes $ic4$.

Now, a question arises: which are the most common trichords and tetrachords? Regarding the common practice period (around 1650 to 1900), the harmonies are mainly built by superimposing thirds on the 7 degrees of the major, harmonic and melodic minor scales [12, 13]. This leads to the 4 basic triads and the 7 basic seventh chords, which correspond to set classes 3-10, 3-11a, 3-11b, 3-12, and 4-19a, 4-19b, 4-20, 4-26, 4-27a, 4-27b, 4-28, respectively. In addition, the augmented sixth chords give rise to the 3-8a (Italian) and 4-25 (French). Table 1 shows all trichords from 3-8 to 3-12 and Table 2 all tetrachords from 4-19 to 4-28. They include the symbols here used to represent them, the intervallic forms (starting from the root) and the interval-class vectors. For other musical styles, such as Pop, Latin or Jazz, all harmonies in these tables are quite common. A useful list of most common chords in these styles is given in [14].

Figures 8 and 9 give the graphic representations of those harmonies (on a rectangular grid), where the roots are marked with circles and the dashed lines represent tritones ($ic6$).

Trichord	Symbol	IF	ICV
3-8a	7*	462	010101
3-8b	Ø**	642	010101
3-9	sus4	525	010020
3-10	dim	336	002001
3-11a	m	345	001110
3-11b	M	435	001110
3-12	+	444	000300

Table 1. Characteristics of trichords 3-8 to 3-12. An asterisk (*) means "omit 5" and a double asterisk (**) "omit ♭3". A major chord (3-11b) is usually represented by the root without any symbol. IF: Intervallic Form, starting from the root. ICV: Interval-Class Vector.

Tetra-chord	Sym-bol	IF	ICV	Basic Trichords
4-19a	mΔ	3441	101310	m, +
4-19b	Δ♯5	4431	101310	M, +
4-20	Δ	4341	101220	m, M
4-21	9*	2262	030201	7*, Ø**
4-22a	(9)	2235	021120	sus4, M
4-22b	m4	3225	021120	sus4, m
4-23	7sus	5232	021030	2 x 4sus
4-24	7♯5	4422	020301	7*, Ø**, +
4-25	7♭5	4242	020202	2 x 7*, 2 x Ø**
4-26	m7	3432	012120	m, M
4-27a	Ø	3342	012111	Ø**, dim, m
4-27b	7	4332	012111	7*, dim, M
4-28	O	3333	004002	4 x dim

Table 2. Characteristics of tetrachords 4-19 to 4-28. An asterisk (*) means "omit 5" and a double asterisk (**) "omit ♭3". Symbol "(9)" means "add 9", whereas symbol "9" adds both the minor seventh and the ninth. IF: Intervallic Form, starting from the root. ICV: Interval-Class Vector.

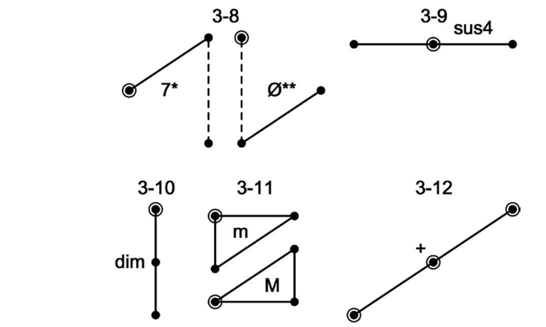

Figure 8. Trichords 3-8 to 3-12 on a rectangular grid.

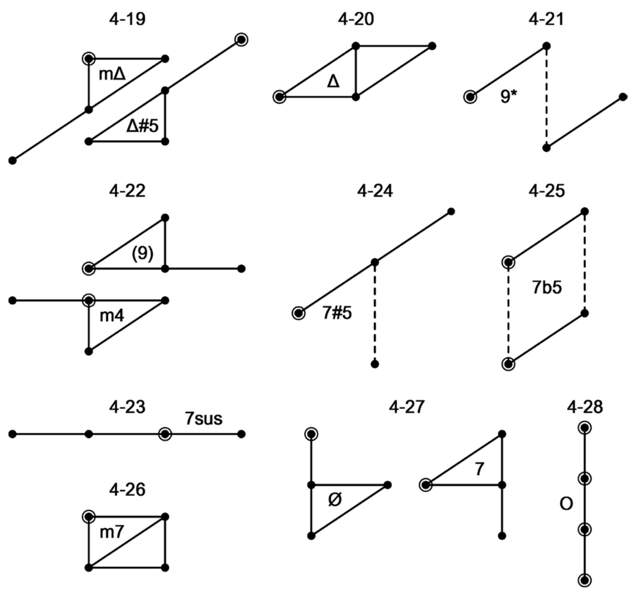

Figure 9. Tetrachords 4-19 to 4-28 on a rectangular grid.

The last column of Table 2 shows which and how many trichords from Table 1 are contained in each tetrachord, which can be easily visualized by comparing Figures 8 and 9. Note that both tetrachords 4-20 and 4-26 contain a major and a minor triad, but there is a great difference between them: 4-20 includes a semitone ($ic1 = 1$) while 4-26 does not ($ic1 = 0$).

Therefore, representing the trichords and tetrachords on the rectangular or polar grids gives us an insight into their inner structures and the relationship among them. Moreover, representing those chords on the Harmonic Wheel, which allows to visualize the relations among the chords belonging to a major or minor scale, will give us a further insight into the characteristics of tonal music, the relationship among the keys and a broader perspective of the theory of modulation.

4. REPRESENTATION OF HARMONIES

Apart from representing single chords, we will look for more general applications. Thus, a pair of examples on diatonic modulation are given, which deal with finding the pivot chords. As well, it is interesting to represent full harmonies from musical excerpts or whole pieces, since they can show the underlying design of the composition. Thus, five examples from different musical styles are represented on the Harmonic Wheel. The corresponding audios are available both on Spotify and iTunes. All representations happened to be simple and compact, and therefore easy to memorize, which makes this representation system an efficient mnemonic notation. Apart from Tables 1 and 2, Table 3 gives the symbols and notes of other extended and altered chords used in the examples, for the root C.

4.1. Diatonic Modulation: Pivot Chords

Diatonic modulation consists in changing from one key to another by means of a common or pivot chord, which is interpreted differently in each key [12, 13]. Finding all pivot chords between the two keys by comparing the chords associated to each of them is laborious. On the contrary, the Harmonic Wheel provides a simple and visual procedure for finding them. For simplicity, we will only consider consonant pivot chords, that is, major or minor.

We will start with the modulation from C Major to G Major. Figure 10 shows the original key with dark blue lines and, superimposed to it, the destination key with light blue lines. For clarity, only the curved rectangles defining the keys are represented. But, in practice, the scale degrees printed on the plastic disc will show the two degrees each chord represents in the two keys. From that figure, it is obvious that there are four common or pivot chords: C, Am, G and Em.

Secondly, let us examine the modulation from D♭ Major to B Major, which will involve enharmonic chords. The two keys are shown in Figure 11 with dark and light blue lines, respectively. The pivot chords are directly obtained from this figure, which are either G♭ and E♭m or F♯ and D♯m.

Chord	Notes	Chord	Notes
C6	C E G A	Cm(9)	C E♭ G D
C(9)	C E G D	Cm9	C E♭ G B♭ D
C9	C E G B♭ D	Cm7/6	C E♭ G A B♭
C7♯9	C E G B♭ D♯	Cm7/11	C E♭ G B♭ F
C9/13	C E G B♭ D A	Cm11	C E♭ G B♭ D F

Table 3. Symbols and notes of some extended and altered chords with root C.

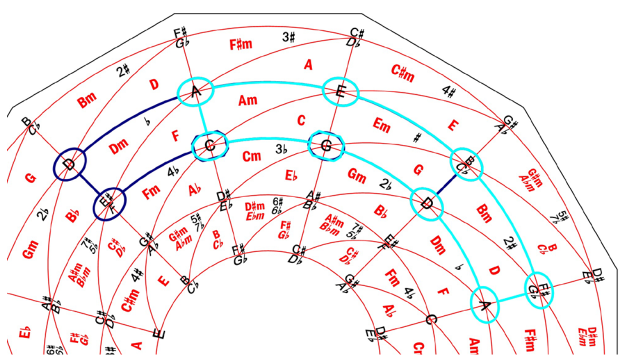

Figure 10. Pivot chords when modulating from C Major to G Major.

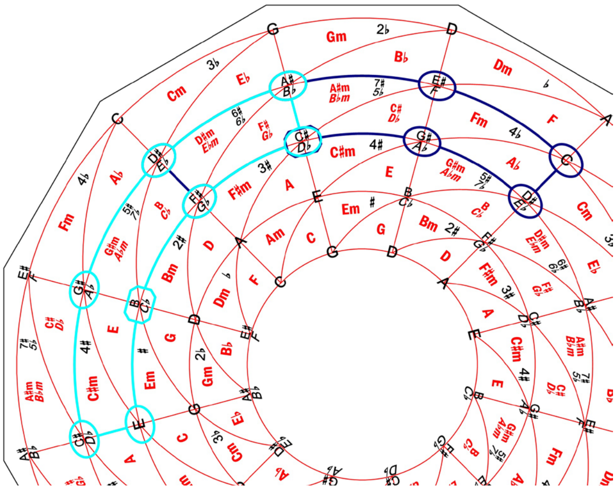

Figure 11. Pivot chords when modulating from D♭ Major to B Major.

4.2. RL operations: Beethoven's Ninth Symphony

The next example is an excerpt from the second movement (Scherzo) of Beethoven's Ninth Symphony. In mm. 143-176, there is a series of consonant triads related by *R* and *L* operations as follows:

C Am F Dm B♭ Gm E♭ Cm A♭ Fm

D♭ B♭m G♭ E♭m C♭ A♭m E C♯m A

Figure 12 shows these triads on the Harmonic Wheel, where they follow a circular pattern on the cycle of fifths (or fourths), which is incomplete but includes both the major and minor triads. In each *R* or *L* operation, two notes remain fixed, which are shown on the interval line being crossed. The same example is analyzed in [7] and represented on a 2D Tonnetz, but in this case the representation takes up a full page due to the length of the chord progression, even though it does not complete an entire cycle.

4.3. Tonal Composition: Autumn Leaves

Autumn Leaves by Kosma [14] is one of the most well-known Jazz Standards and one that Jazz students first learn. Its harmony repeats the following chord sequence:

Cm7 F7 B♭Δ E♭Δ A$^{\text{Ø}}$ {D7} Gm (G7)

All these chords belong to the same key, B♭ major or G natural minor, with the only exception of D7, which for that reason is written in braces. The last chord, G7 in parentheses, does not belong to that key either, but it is included only sometimes to resolve to Cm7. Except Gm, all chords are seventh chords, so they cannot be represented properly on the Tonnetz (since it just contains triads). On the contrary, they largely match the chord types indicated on the Harmonic Wheel when choosing the B♭ major scale (Figure 13). The chord Dm7 is substituted with D7 to resolve to Gm. As well, there is a last chord sequence, slightly different from the previous one:

{C9 Fm7 B♭7} E♭Δ A$^{\text{Ø}}$ {D7♯5} Gm (G7)

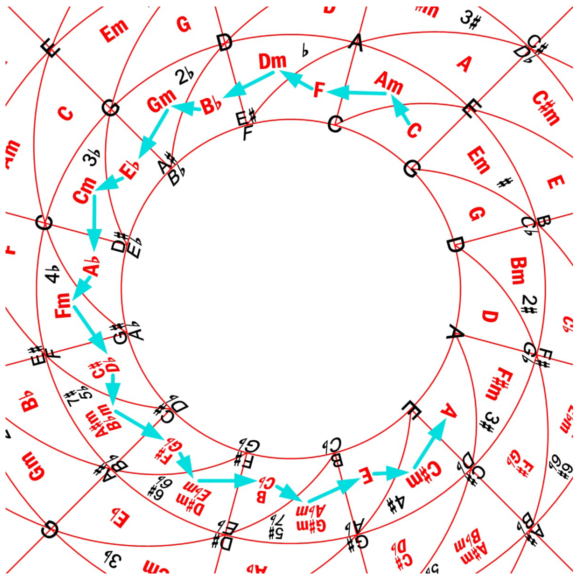

Figure 12. Harmonic structure of Beetheven's Ninth Symphony, second movement, mm. 143-176.

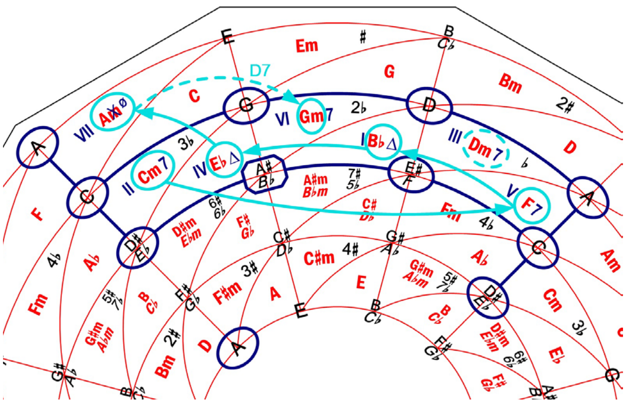

Figure 13. Harmonic structure of Autumn Leaves.

where the chords that are different are in braces.

This song is, in some cases, studied in the 12 keys [15], so using the Harmonic Wheel is of great help to visualize and memorize the harmonies.

4.4. Chords by Major Thirds: Giant Steps

In contrast to the last example, *Giant Steps* by Coltrane [16] is considered a challenging Jazz tune, since its harmony does not belong to any major or minor key. On the contrary, it follows a major third cycle, that is, a spiral line on the Harmonic Wheel. The full chord sequence is the following:

BΔ [D7] GΔ [B♭7] E♭Δ

[Am7 D7] GΔ [B♭7] E♭Δ [F♯7] BΔ

[Fm7 B♭7] E♭Δ [Am7 D7] GΔ [C♯m7 F♯7] BΔ

[Fm7 B♭7] E♭Δ [C♯m7 F♯7]

This harmony is based on 3 major seventh chords: BΔ, GΔ and E♭Δ, whose roots are a major third apart, thus dividing the octave into 3 equal parts. If we consider those chords as I degrees, the chords before them are either a V7 or a pair IIm7 V7, which are written in brackets to simplify the analysis. The cadence IIm7 V7 IΔ is the same as the first 3 chords in our last example (Cm7 F7 B♭Δ), which has a clear representation on the Harmonic Wheel. The other cadence, V7 IΔ, is simply a reduction of that one.

Figure 14 shows the harmonic structure of this piece, where the 3 main chords (BΔ, GΔ and E♭Δ) follow a spiral line and complete a major third cycle. The cadences V7 IΔ and IIm7 V7 IΔ are represented by solid and dashed lines, respectively. For clarity, the first part of the song is represented in blue and the second one in green. Of course, the 3 major chords are assumed to be major seventh chords.

As in the previous example, this song is also studied in the 12 keys [17], so the Harmonic Wheel is again a helpful tool, once the diagram in Figure 14 has been memorized.

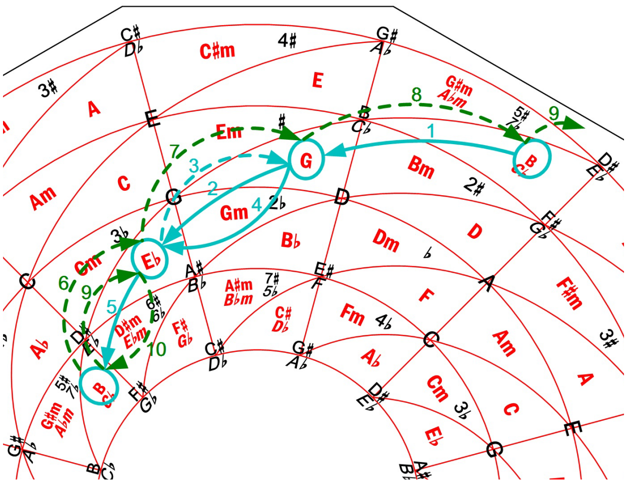

Figure 14. Harmonic structure of Giant Steps.

4.5. Chords in Béla Bartók's Axes: Indudable

Béla Bartók's axis system was first published by one of his disciples, Ernö Lendvai, after performing an exhaustive analysis of his work [18]. In summary, it states that relative and parallel chords have the same harmonic function (tonic, subdominant or dominant). This leads to groups of 8 chords with the same harmonic function, their roots being a minor third apart; that is, they follow a minor third cycle or a radius on the Harmonic Wheel. For example,

C Am A F♯m F♯ E♭m E♭ Cm

Indudable by Nuño [19] is a Bossa Nova whose second section has a harmony based on 2 such axes, one with major chords and the other with minor chords. The following basic chord sequence is played four times:

G♯m C♯ Fm B♭ Dm G Bm E

The major chords C♯, B♭, G, E belong to one of the axes and the minor chords G♯m, Fm, Dm, Bm to the other one. If the first chord, G♯m, is considered a Im degree, the next chord is the major IV degree (as in a melodic minor scale), C♯, which is enharmonic to D♭. Then, if this chord is now considered a new I degree, then the next chord is the IIIm degree (as in a major scale), Fm. And this process is repeated cyclically. The corresponding diagram is represented in Figure 15, where the relationship among the chords, as well as the minor third cycles in the radial direction followed by them, are clearly shown.

The real chords, however, are more complex, since they contain 3 to 6 notes to enrich the harmony. The actual chord sequence is given below and is played twice. For clarity, similar chords have been grouped in brackets.

[G♯m(9) G♯m7/6] [C♯ C♯Δ] [Fm(9) Fm9] [B♭ B♭Δ]

[Dm7 ⁄] [G6 ⁄] [Bm7 ⁄] [E7sus ⁄]

[G♯m(9) G♯m7/6] [C♯^{O} ⁄] [Fm(9) Fm7/6] [B♭^{O} ⁄]

[Dm11 ⁄] [G6 ⁄] [Bm7 Bm7/11] [E7♭5 ⁄]

Each chord lasts one beat and the symbol "⁄" means to repeat the last beat (in these cases, the previous chord). As well, two diminished-seventh chords (C♯^{O} and B♭^{O}) were included to increase the variety of chord types.

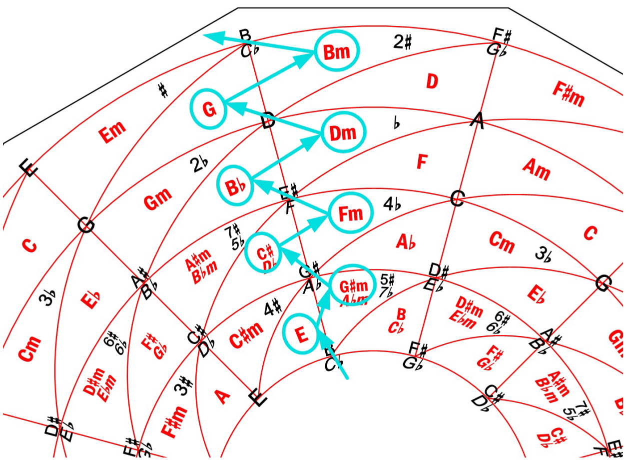

Figure 15. Harmonic structure of Indudable, second section.

4.6. A Piece with Modulations: All The Things You Are

All The Things You Are by Kern [14] is another well-known Jazz song. An Intro consisting of chords D♭7♯9 and C7♯9 is followed by this harmony:

Fm7 B♭m7 E♭7 A♭Δ D♭Δ [Dm7 G7] CΔ ⁒

Cm7 Fm7 B♭7 E♭Δ A♭Δ [A$^{\varnothing}$ D7] GΔ ⁒

Am7 D7 GΔ ⁒ F♯$^{\varnothing}$ B7 EΔ {C7♯5}

Fm7 B♭m7 E♭7 A♭Δ D♭Δ {G♭9/13}

Cm7 {B^{O}} B♭m7 E♭7 A♭6 (G$^{\varnothing}$ C7)

where each chord lasts one measure, as well as each pair in brackets or parentheses, and the symbol "⁒" means to repeat the last measure (in these cases, the previous chord).

The chords in the first phrase belong to the same key, A♭ major or F natural minor, except the last three, which form a cadence ending in CΔ, the dominant of F minor. Figure 16 shows these harmonies, where the first chord types are exactly as indicated on the plastic disc.

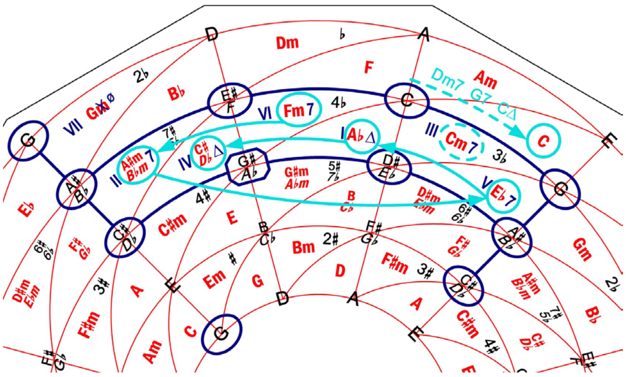

Figure 16. Harmonic structure of All The Things You Are, first phrase.

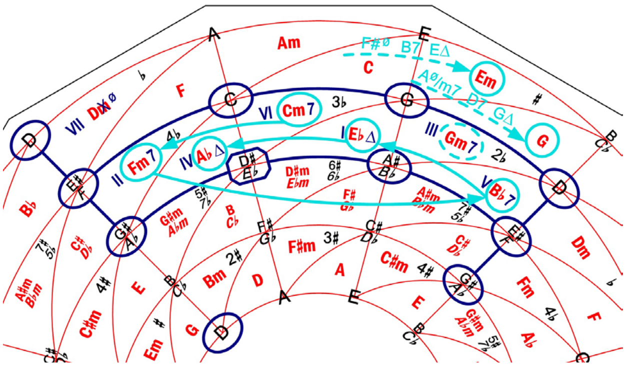

Figure 17. Harmonic structure of All The Things You Are, second phrase.

The chords in the second phrase are analogous to those in the first one, but in E♭ major or C natural minor, which represents a (transient) modulation to the dominant. The new chords are found by simply rotating the plastic disc one step clockwise, as seen in Figure 17. Again, the first chord types are exactly as indicated on the plastic disc. Unlike the first phrase, now the cadence starts with a half-diminished chord, $A^{Ø}$, but then it is repeated starting with Am7. Another cadence follows, ending in EΔ, and moving to C7♯5 to resolve to Fm7, the original key.

The rest of chords, with two exceptions written in braces, belong to A♭ major or F natural minor, so they are found by rotating the plastic disc one step counterclockwise. Now, the Cm7 chord is included and the phrase ends in A♭6, the tonic chord with the added major sixth. The last two chords in parentheses are used to return to the beginning.

As shown in Figures 16 and 17, most chords in this song are diatonic to a particular key, so the chord types are exactly as indicated on the Harmonic Wheel. Moreover, most of the time, their roots move by descending fourths, thus being easy to visualize and memorize.

5. CONCLUSIONS

The Harmonic Wheel is a physical tool consisting of two rotating discs: one cardboard, with a Tonnetz transformed into a polar grid, and the other plastic, with the lines defining the major, harmonic and melodic minor scales, together with the scale degrees and the symbols of the corresponding seventh chords. It has been used to represent single chords, excerpts and full harmonies from different musical styles, considering both triads and seventh chords, and including modulations. In some cases, the harmonies followed one of the three cycles defined by the consonant intervals and, in others, they were diatonic to a particular key. In all cases, the Harmonic Wheel has proved to be a powerful and versatile tool for representing the harmonies and, therefore, to show the underlying structures of the musical compositions here considered. Furthermore, it provides an efficient mnemonic notation by means of simple and compact diagrams.

6. REFERENCES

1. A. S. Crans, T. M. Fiore and R. Satyendra, “Musical Actions of Dihedral Groups,” *American Mathematical Monthly*, vol. 116, no. 6, pp. 479-495, 2009.

2. J. Douthett and P. Steinbach, “Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition,” *Journal of Music Theory*, vol. 42, no. 2, pp. 241-263, 1998.

3. E. Gollin, “Some Aspects of Three-Dimensional ‘Tonnetze’,” *Journal of Music Theory*, vol. 42, no. 2, pp. 195-206, 1998.

4. D. Tymoczko, “The Generalized Tonnetz,” *Journal of Music Theory*, vol. 56, no. 1, pp. 1-52, 2012.

5. L. A. Piovan, “A Tonnetz Model for Pentachords,” *Journal of Mathematics and Music*, vol. 7, no. 1, pp. 29-53, 2013.

6. V. Mohanty, “A 5-Dimensional Tonnetz for Nearly Symmetric Hexachords,” *Journal of Mathematics and Music*, DOI: 10.1080/17459737.2020.1799087, 2020.

7. R. Cohn, “Neo-Riemannian operations, parsimonious trichords, and their ‘Tonnetz’ representations,” *Journal of Music Theory*, vol. 41, no. 1, pp. 1-66, 1997.

8. S. B. P. Briginshaw, “A Neo-Riemannian Approach to Jazz Analysis,” *Nota Bene: Canadian Undergraduate Journal of Musicology*, vol. 5, no. 1, pp. 57-87, 2012.

9. L. Nuño, *Harmonic Wheel*. Valencia, Spain: Luis Nuño, 2008.

10. A. Forte, *The Structure of Atonal Music*. New Haven, CT: Yale University Press, 1973.

11. L. Nuño, “A Detailed List and a Periodic Table of Set Classes,” *Journal of Mathematics and Music*, DOI: 10.1080/17459737.2020.1775902, 2020.

12. A. Schönberg, *Theory of Harmony*. 3rd ed. Berkeley, CA: University of California Press, 1983.

13. W. Piston, *Harmony*. 5th ed. New York: W. W. Norton & Co, 1988.
14. C. Sher, *The New Real Book, Vol. 1*. Petaluma, CA: Sher Music Co., 1988.
15. J. Aebersold, *Vol. 67: Tune Up - Standards in All 12 Keys*. New Albany, IN: Jamey Aebersold Jazz, Inc., 1995.
16. C. Sher, *The New Real Book, Vol. 1I*. Petaluma, CA: Sher Music Co., 1991.
17. J. Aebersold, *Vol. 68: Giant Steps - Standards in All 12 Keys*. New Albany, IN: Jamey Aebersold Jazz, Inc., 1995.
18. E. Lendvai, *Béla Bartók. Un Análisis de su Música (Béla Bartók. An Analysis of his Music)*. Barcelona, Spain: IDEA BOOKS, S. A., 2003.
19. L. Nuño, *Puesta de Sol, Vol. 1*. Madrid, Spain: Acordes Concert S. L., 2012.